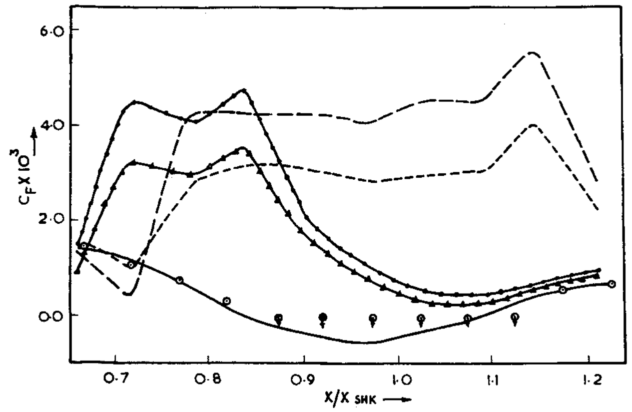

Fig. 3 Skin-friction distribution.

Results

The typical case considered is for a freestream Mach number of 2.0, a Reynolds number of 0.296×10^6 based on the distance X_{SHK} from the leading edge to the shock impingement point, and an incident shock angle of 32.585 deg.[4] For this set of data, the shock is strong enough (pressure ratio = 1.4) to trigger separation. The computation was done for five cases: 1) no suction along the wall, 2) normal suction, $\phi = 90$ deg at the location from $X/X_{SHK} =$ 0.7817 to 1.1569, 3) vectored suction, $\phi = 45$ deg at the locations as in case 2, 4) normal suction from $X/X_{SHK} = 0.7192$ to 0.8442, and 5) vectored suction at the same locations as in case 4.

The computed surface pressure distribution in the interaction region is presented in Fig. 2. For the vectored upstream suction case 5, the pressure jump is closes to the inviscid flow conditions case and reaches its postshock value quite smoothly. It is interesting to note that the pressure rise is steeper than that corresponding to normal suction. Though the pressure plateau indicating separation has vanished for all the examples in which suction is considered, the upstream vectored suction has a minimum effect in the downstream direction (Fig. 3).

This study indicates that the upstream vectored suction not only eliminates separation but that its influence in the neighborhood is limited. If the prior knowledge of separation bubble location is not available or some minor changes in the input data are required, a judicious choice of upstream vectored suction can control the flow effectively. However, a detailed assessment regarding the locations, rates, and angles, for such suction should be carried out to optimize its fruitful usage.

References

[1]Tassa, Y. and Sankar, N. L., "Effect of Suction on the Shock-Separated Boundary Layer—A Numerical Study," *AIAA Journal*, Vol. 17, Nov. 1979, pp. 1268-1270.

[2]MacCormack, R. W. and Baldwin, B. S., "A Numerical Method for Solving the Navier-Stokes Equations with Application to Shock-Boundary Layer Interaction," AIAA Paper 75-0001, Jan. 1975.

[3]Purohit, S. C., Shang, J. S., and Hankey, W. L., "Effect of Suction on the Wake Structure of a Three-Dimensional Turret," AIAA Paper 83-1738, July 1983.

[4]Hakkinen, R. J., Greber, I., Trilling, L., and Abarbanel, S. S., "The Interaction of Oblique Shock Wave with a Laminar Boundary Layer," NASA Memo 2-18-59W, March 1959.

Rapid Computation of Unsteady Transonic Cascade Flows

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Introduction

REFERENCE 1 describes a method for computing the unsteady flow through a cascade for an interblade phase angle using only one set of computational data. This computational data is calculated for one blade moving with the other blades held stationary. Further simplification is possible because only the indicial response of the cascade needs to be calculated; from this result the flow for any time-dependent motion can be constructed by superposition. For subsonic or supersonic flows, the indicial method is described by Lomax,[2] the method was developed for transonic flows by Ballhaus and Goorjian[3] and Nixon.[4] Nixon shows that unsteady pressure distributions can be computed for transonic flows and that, to a first-order approximation in shock motion, the lift and pitching moments are linear functions of amplitude. Because of the need to compute only one transonic flow solution in order to obtain a result for any interblade phase angle and motion, the method developed in Ref. 1 is very computationally efficient.

The main difficulty in constructing the indicial solution for the cascade is that a reasonable number of fixed blades must be represented in the calculation. The definition of reasonable will be discussed later. In the first attempt[5] to compute examples using the method of Ref. 1, seven blades were used in the computation, and the results for a flat plate cascade in subsonic flow did not agree well with the results of Verdon and Casper.[6] The most obvious difference was the absence of resonance in the indicial theory result. It was speculated that the disagreement resulted from insufficient blades in the cascade. It is obvious that if a large number of blades are required in the cascade, the computational efficiency of the method is greatly reduced, since the indicial calculation could require computer resources equivalent to that used to calculate cascade flows for a variety of interblade phase angles. In this Note a method of including a large number of blades in the calculation without incurring a heavy computational requirement is described. Results that are a considerable improvement on those described in Ref. 5 are given. The analysis contained herein is for an unstaggered cascade.

Analysis

Although the indicial theory described in Ref. 4 can give pressure distributions, only lift and pitching moments are considered here for reasons of clarity in presentation. The gap/chord ratio is denoted by h and the freestream Mach number by M_∞.

The lift given by the theory in Ref. 1 is

$$C_L(t) = \sum_{n=-\infty}^{\infty} C_{L_n}(t - n\sigma) \tag{1}$$

Received July 24, 1986.

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where C_{L_n} is the lift induced at blade zero by the motion of the nth blade with the other blades stationary and σ the interblade phase angle. If the flow is harmonic in time, with frequency ω,

$$C_L(t) = \bar{C}_L e^{i\omega t}, \qquad C_{L_n}(t-n\sigma) = \bar{C}_{L_n} e^{i\omega(t-n\sigma)} \tag{2}$$

In a cascade, the unsteady flow induced on blade zero by the nth blade can be characterized by the asymptotic form of the unsteady flow, provided that the nth blade is sufficiently far from blade zero (for example, several chord lengths). The asymptotic behavior for the induced normal velocity is (Appendix A)

$$v(x,y) \approx A \frac{e^{-i\bar{K}|y|}}{|\bar{y}|^{1/2}} \tag{3}$$

where A is a complex constant and $\bar{y} = (1-M_\infty^2)^{1/2} y$; K is defined in Appendix A. This result is obtained from classic subsonic airfoil theory (see Ref. 7). The lift induced on a blade is proportional to the induced upwash parameter, which is given by Eq. (3). Hence it follows that

$$C_{L_n} = C_{L_m} \left| \frac{m}{n} \right|^{1/2} e^{-iK(|n|-m)\bar{h}} \tag{4}$$

where the relation $\bar{y} = n\bar{h}$ is used, and m is a given blade beyond which Eq. (3) is a good approximation. It is assumed that C_{L_m} is known. Note that the actual value of $v(x,y)$ is not given by Eq. (3) since this result only holds if there are no other blades. However, the actual upwash will vary with y in a form similar to Eq. (3). Using Eqs. (1), (2), and (4) gives

$$\bar{C}_L = \sum_{n=-m+1}^{m-1} \bar{C}_{L_n} e^{-in\sigma} + \sum_{n=m}^{\infty} \bar{C}_{L_m} \left(\frac{m}{n}\right)^{1/2} \times \exp[-iK(n-m)\bar{h} - in\sigma) + \sum_{n=-\infty}^{-m} \bar{C}_{L_m} \left(\frac{m}{n}\right)^{1/2} \times \exp[-iK(n-m)\bar{h} - in\sigma] \tag{5}$$

It can be seen that the effect of the cascade blades diminishes as $n^{-1/2}$, and hence a large number of blades is necessary in the initial computation of C_{L_n} unless the approximation of Eq. (4) is used. It is also of interest to further examine Eq. (5). The two infinite sums are bounded (see Appendix B) unless

$$\pm K\bar{h} - \sigma = 2p\pi \tag{6}$$

where p is an integer. Thus, for an interblade phase angle given by

$$\sigma = \pm K\bar{h} - 2p\pi \tag{7}$$

$\bar{C}_L$ will be infinite. For the flat plate cascade considered by Verdon and Caspar[6] with $M_\infty = 0.5$, $h = 1$ and the reduced frequency ν equated to unity. Equation (7) gives for $p = 0$

$$\sigma = \pm 33.1 \text{ deg} \tag{8}$$

at which angle the lift and moment coefficients will become infinite. This value of σ corresponds to that given by Verdon and Casper[6] for resonance.

It should be noted that this proof requires that there be an infinite number of blades in the cascade, which implies that a finite cascade will not undergo resonance. If the cascade is assumed to be a valid approximation to an actual compressor with N blades, then this can be regarded as an infinite cascade with the interblade phase angle being integer multiples of the basic angle $2\pi/N$. In this case, resonance could occur for certain combinations of K and $\bar{h}$ which are themselves functions of M_∞, ν, and h.

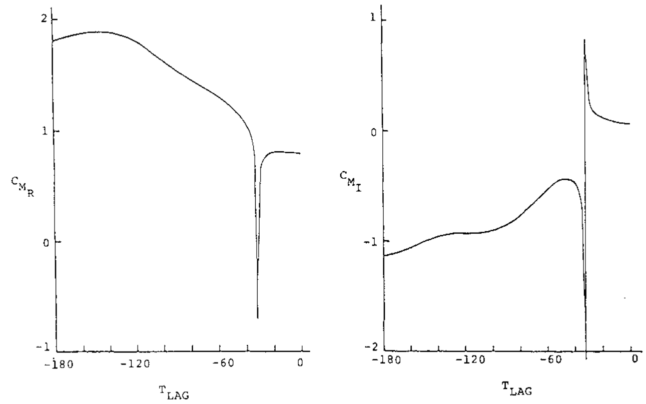

Fig. 1 Moment due to torsion for a flat plate; $M_\infty = 0.5$, $\nu = 1.0$, $h = 1.0$.

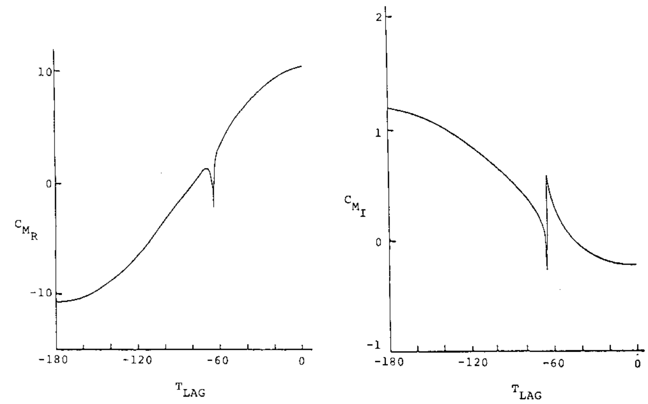

Fig. 2 Moment due to torsion for a NACA-0012 section; $M_\infty = 0.75$, $\nu = 1.0$, $h = 2$.

Results for a flat plate cascade at $M_\infty = 0.5$, $\nu = 1.0$, and $h = 1.0$ computed using Eq. (5) are shown in Fig. 1. In this calculation, the total number of blades in the numerical calculation is 13 and $m = 3$. The number of blades used in the sum in Eq. (5) is 100. In Fig. 2 a result for a transonic case is shown, namely a cascade with a NACA-0012 section, $M_\infty = 0.75$, $h = 2.0$, and $\nu = 1.0$. It can be seen that there is a considerable difference between a subsonic case and a transonic case.

Concluding Remarks

A method of rapidly computing unsteady transonic cascade flows has been examined and extended. It is shown that resonance can occur for an infinite cascade and that there is a significant difference between subsonic and transonic flows.

Appendix A: Velocity Induced by a Moving Blade

The object is to estimate the induced upwash at some station y due to the oscillatory motion of a blade at $y = 0$. It is assumed that y is sufficiently far from the oscillating blade that linear theory can apply.

From classic subsonic theory, the velocity potential in the far field at a reduced frequency ν is given by (see Ref. 7)

$$\phi(x,\bar{y}) = \frac{\bar{A}}{(Kr)^{1/2}} \exp\left[-i\left(Kr - \frac{\pi}{4}\right)\right] e^{iM_\infty^2 \Omega x} \tag{A1}$$

where

$$r = (x^2 + \bar{y}^2)^{1/2} \tag{A2}$$

$$\Omega = \nu/(1-M_\infty^2), \quad K = M_\infty \Omega, \quad \bar{y} = (1-M_\infty^2)^{1/2} y \tag{A3}$$

Far from the moving blade, x can be taken as zero, and on differentiation with respect to y,

$$v(0,\bar{y}) = \frac{\bar{A}}{(1-M_\infty^2)^{1/2}} \frac{e^{i\pi/4}}{K^{1/2}} \left(\frac{-1}{2\bar{y}^{3/2}} - \frac{iK}{\bar{y}^{1/2}} \right) e^{-iK\bar{y}} \tag{A4}$$

As $\bar{y} \to \infty$, $v(0,\bar{y})$ can be approximated by

$$v(0,\bar{y}) = \frac{-i\bar{A}}{(1-M_\infty^2)^{1/2}} \frac{K^{1/2}}{\bar{y}^{1/2}} e^{-iK\bar{y}} e^{i\pi/4} \tag{A5}$$

$$= A \frac{e^{-iK\bar{y}}}{\bar{y}^{1/2}} \tag{A6}$$

where $\bar{A}$ is a complex constant.

Appendix B: Convergence of the Series

The convergence of the series $\Sigma_{n=1}^{\infty} (e^{in\alpha}/n^{1/2})$ is due primarily to the cancellation effects provided by the term $e^{in\alpha}$, as the series $\Sigma_{n=1}^{\infty} (1/n^{1/2})$ is divergent, and therefore $\Sigma_{n=1}^{\infty} (e^{in\alpha}/n^{1/2})$ is not absolutely convergent. For values of α for which an integer p exists such that $p\alpha = \pi$, the series $\Sigma_{n=1}^{\infty} (e^{in\alpha}/n^{1/2})$ can be reduced to an alternating series by grouping all consecutive terms of the same sign; the series that follows has monotonically decreasing terms and is therefore convergent.

While a somewhat similar approach can be followed for an arbitrary α, the procedure tends to become quite complex. The following proof of convergence, on the other hand, is more direct and exploits equally well the alternating character of $e^{in\alpha}$:

$$\sum_{n=1}^{\infty} \frac{e^{in\alpha}}{n^{1/2}} = e^{i\alpha} + \frac{e^{i2\alpha}}{2^{1/2}} + \ldots + \frac{e^{in\alpha}}{n^{1/2}} + \ldots$$

$$= \left(1 - \frac{1}{2^{1/2}}\right) e^{i\alpha} + \left(\frac{1}{2^{1/2}} - \frac{1}{3^{1/2}}\right)(e^{i\alpha} + e^{i2\alpha})$$

$$+ \left[\frac{1}{n^{1/2}} - \frac{1}{(n+1)^{1/2}}\right](e^{i\alpha} + \ldots + e^{in\alpha}) + \ldots$$

or

$$\sum_{n=1}^{\infty} \frac{e^{in\alpha}}{n^{1/2}} = \sum_{n=1}^{\infty} \left[\frac{1}{n^{1/2}} - \frac{1}{(n+1)^{1/2}}\right](e^{i\alpha} + \ldots + e^{in\alpha})$$

The series on the right-hand side of the preceding identity is absolutely convergent since

$$\frac{1}{n^{1/2}} - \frac{1}{(n+1)^{1/2}} = \frac{1}{n^{1/2}}\left[1 - 1\bigg/\left(1 + \frac{1}{n}\right)^{1/2}\right]$$

$$\leq \frac{1}{n^{1/2}}\left[1 - 1\bigg/\left(1 + \frac{1}{n}\right)\right]$$

$$= \frac{1}{n^{1/2}}\left(\frac{1}{1+n}\right)$$

$$< \frac{1}{n^{3/2}}$$

and hence

$$\left| \left[\frac{1}{n^{1/2}} - \frac{1}{(n+1)^{1/2}}\right](e^{i\alpha} + \ldots + e^{in\alpha}) \right| < \frac{1}{n^{3/2}} \left| \frac{1 - e^{in\alpha}}{1 - e^{i\alpha}} \right|$$

$\Sigma_{n=1}^{\infty} (1/n^{3/2})$ is convergent (by Cauchy's integral test), and

$$\left| \frac{1 - e^{in\alpha}}{1 - e^{i\alpha}} \right| = \frac{\sin^2 n\alpha/2}{\sin^2 \alpha/2} \leq \frac{1}{\sin^2 \alpha/2}$$

finite and independent of n, for $\alpha \neq 0, 2p\pi \ldots$

It follows that the series

$$\sum_{n=1}^{\infty} \left[\frac{1}{n^{1/2}} - \frac{1}{(n+1)^{1/2}}\right](e^{i\alpha} + \ldots + e^{in\alpha})$$

is convergent and hence $\Sigma_{n=1}^{\infty} (e^{in\alpha}/n^{1/2})$ is convergent.

Acknowledgment

The work reported here was funded by the Naval Air Systems Command under Contract N00014-83-C-0435.

References

[1]Nixon, D., "Computation of Unsteady Transonic Cascade Flows," *AIAA Journal*, Vol. 21, May 1983, pp. 790-792.

[2]Lomax, H., "Indicial Aerodynamics," *AGARD Manual of Aeroelasticity*, Vol. II, Ch. 6, 1960.

[3]Ballhaus, W. F. and Goorjian, P. M., "Computation of Unsteady Transonic Flow by the Indicial Method," *AIAA Journal*, Vol. 16, Feb. 1978, pp. 117-124.

[4]Nixon, D., "Notes on the Transonic Indicial Method," *AIAA Journal*, Vol. 16, June 1978, pp. 614-616.

[5]Nixon, D., "Prediction Methods for the Unsteady Transonic Flow through a Cascade," Nielsen Engineering and Research, TR 306, 1983.

[6]Verdon, J. M. and Caspar, J. R., "Subsonic Flow Past an Oscillating Cascade with Finite Mean Flow Deflection," *AIAA Journal*, Vol. 18, May 1980, pp. 540-548.

[7]Nixon, D., "An Airfoil Oscillating at Low Frequencies in a High Subsonic Flow," British ARC CP 1285, 1974.

Numerical Examination of a Tube- and Disk-Type Combustor Configuration

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Introduction

An experimental tube and disk combustor configuration was examined numerically. This configuration is a compromise between the disk-in-duct configuration,[1] the dump ramjet burner,[2] and the prevaporizing/premixing fuel feed-tube-type turbojet. The tube and disk configuration models

Presented, in part, as Paper 82-1074 at the AIAA/ASME/SAE 18th Joint Propulsion Conference, Cleveland, OH, June 21-23, 1982; received Dec. 22, 1984; revision submitted June 26, 1986.

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